A Measurement Allocation Scheme for Reliable Data Gathering in Spatially Correlated Sensor Networks

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Abstract—In this paper, we consider the measurement allocation problem in a spatially correlated sensor field. Our goal is to determine the probability of each sensor’s being measured based on its contribution to the estimation reliability; it is desirable that a sensor improving the estimation reliability is measured more frequently. We consider a correlation model reflecting transmission power limit, noise in measurement process and channel, and channel attenuation. Then the estimation reliability is defined as the distortion error between the event source in the sensor field and its estimation at the sink. Motivated by the correlation nature, we model the measurement allocation problem into a cooperative game, and then express each sensor’s contribution using Shapley value - a formal quantification of individual player’s average marginal contribution. Against the intractability in the computation of exact Shapley value, we deploy randomized method that enables to compute approximate Shapley value within reasonable time. In numerical experiments, we evaluate approximate Shapley value by comparing it to the exact one, and illustrate that measurement allocation according to Shapley value turns to the balance between the estimation reliability and network lifetime.

I. INTRODUCTION

In wireless sensor networks (WSNs), exploiting the correlation nature of a spatial phenomenon can lead to a significant performance improvement of communication protocol, e.g., efficient information aggregation and prolonged network lifetime. With considering a sensor field wherein phenomena are spatially correlated, there is a principle that the level of correlation differs location by location, and which has been exploited in several different research contexts: i) placement (or localization) [1]–[3]; ii) selection (or activation) [4]–[10]; iii) density decision [11]–[13]; iv) measurement allocation (or observation allocation) [14], [15]; v) power or rate allocation [9], [16]–[19]; vi) considering multi-hop [20], [21].

In this paper, we consider the spatial correlation in an inaccessible sensor field (e.g., enemy line in a battlefield or contaminated area by radioactive fallout) wherein all sensors are cannoned or airdropped randomly. In such a sensor field, the degree of contribution of each sensor differs according to the location of the event source and each sensor’s own location, it is therefore essential to distribute the observation exactly fairly in proportion to the contribution given by each sensor. Inspired by [19] and [4], which have dealt with the problems of maximizing the estimation reliability and minimizing the number of active sensors in spatially correlated sensor fields, respectively, our work focuses on quantifying the contribution of every sensor in terms of the estimation reliability, and allocating the measurements in proportion to the quantified contributions, namely, fair measurement allocation. Considering transmission power limit in each sensor, noise in measurement process and channel, and channel attenuation, it is clear that better estimation reliability is yielded by measuring sensors that are less correlated with one another but highly correlated with the event source [4].

The correlated nature encourages us to model the problem into a cooperative game, and quantify each sensor’s contribution using coalition value. For this, we define the characteristic function as the inverse of the distortion error between an event source in a sensor field and its estimation at sink. A characteristic function in a cooperative game corresponds to the amount of utility achieved by cooperation; it is desirable to apportion the achieved utility to each player in proportion to its average marginal contribution in the cooperation. In this paper, we employ Shapley value [22] to quantify individual sensor’s average marginal contribution. In the context of cooperative game theory, a player’s Shapley value gives an indication of its prospects of playing the game - the higher Shapley value it has, the better it prospects. Intuitively, mutually less correlated sensors observe less correlated data (i.e., the correlation coefficient between sensors affects the distortion positively [4]). In [4], the authors have deployed vector quantization and Voronoi diagram for selecting mutually less correlated sensors.

Unlike general cooperative games, players (i.e., sensors) in our games are not self-interested; finding a set of stable payoff allocations such as core, with which all the players are satisfied, is not of our interest. That is, we are interested in the...
payoff allocation through Shapley value, and not interested in
the finding the core since the players (sensors) are not selfish
in our game. Nonetheless we model our problem into a game
since cooperative game theory also investigates fair division
of the resources, and Shapley value serves this purpose [23].

In spite of those desirable properties, Shapley value has one
major drawback: in many cooperative games, even in simple
games, computing exact Shapley value is intractable [24]; it
is proved that finding exact Shapley value is \#P-complete
(Sharp-P-complete). However only a few research efforts can
be found where approximations are developed to estimate it.
From among them, we use one of the latest approximation
methods based on the randomized method [25].

The remainder of this paper is organized as follows. In
Section II, we introduce the sensor correlation model and
necessary preliminaries about the cooperative game theory
and Shapley value. In Section III, we develop the measurement
allocation game within distortion error criterion. In Section IV,
we present the randomized method. In Section V, we present
some numerical evaluation results, and we conclude this paper
in Section VI.

II. PRELIMINARIES

This section begins with the presentation of the sensor data
correlation model used in this paper. Our model describes
the information structure collected in a spatially
 correlated sensor field considering limited transmission power,
measurement and channel noise, and channel attenuation. Then
we give essential background encompassing cooperative game
theory and Shapley value.

A. Data Correlation Model

In pursuit of [19] and [4], the estimation reliability can be
defined in the mean square error (MSE) criterion: the MSE
between an event source in a sensor field and its estimation at
sink node. Fig.1 illustrates the sensor correlation model. Let
U and \( \hat{U} \) denote an event source and its estimation by sink,
respectively. Then the distortion error between the source and
its estimation is given by

\[
D_E = E \left[ \left( U - \hat{U} \right)^2 \right].
\]  

\( W_i \) indicates sensor i’s observation on U, which is assumed
to be spatially correlated with other sensors’ observations, and
is given as joint Gaussian random variables drawn from

\[
E[W_i] = 0 \quad \text{and} \quad \text{var} \{W_i\} = \sigma_{W_i}^2 \quad \forall i \in N.
\]

Then \( K(i,j) = E[W_iW_j] \) holds where \( K(i,j) \) is a covari-
ance matrix. We consider isotropic covariance matrix [1] that
emphasizes the weak dependencies (i.e. variables that are far
apart are actually often independent). We let \( \gamma = \alpha \|i - j\|/2 \).

If \( \gamma < 2\pi \) for \( \alpha > 0 \),

\[
K(i,j) = \frac{(2\pi - \gamma)(1 + (\cos(\gamma))/2 + \frac{3}{2}\sin(\gamma))}{3\pi}.
\]

and zero otherwise. \( Z_i \) and \( n_i \) denote the measurement noise
and channel noise, respectively, and drawn from \( i.i.d \sim N(0, \sigma_i^2) \) and \( i.i.d \sim N(0, \sigma_n^2) \).

Then the received signal by the sink from sensor i is given by

\[
\hat{W}_i = \sqrt{\frac{P_i}{\sigma_{W_i}^2 + \sigma_Z^2}} h_i (W_i + Z_i) + n_i, \quad \forall i \in N
\]

where \( P_i \) and \( h_i \) are the allocated transmission power and
the channel attenuation coefficient for sensor i, respectively. We
assume that the channel state is detected at each sensor, and
its result is transmitted to the fusion center, and the detected
channel state doesn’t change until the end of transmission.
Moreover, as done in [19], we premise that the sensors are
measured one by one, which implies non-interfered sensor
transmission. Let \( \hat{U}(S) \) be the estimate of \( U \) when only a
subset of the sensors \( S \subseteq N \) send the information, and given by

\[
\hat{U}(S) = \frac{1}{|S|} \sum_{i \in S} \hat{W}_i.
\]

Also (1) is rewritten in terms of a subset \( S \) as

\[
D_E(S) = E \left[ \left( U - \hat{U}(S) \right)^2 \right].
\]

Accordingly, by (2), (4), (5), and (6), the distortion function
\( D_E(S) \) is yielded as

\[
D_E(S) = \sigma_{W_i}^2 - \frac{2}{|S| \sqrt{\sigma_{W_i}^2 + \sigma_Z^2}} \sum_{i \in S} P_i h_i K(U,i) + \frac{1}{|S|^2} \sum_{i \in S} P_i h_i^2 + \frac{1}{|S|^2 (\sigma_{W_i}^2 + \sigma_Z^2)} \sum_{i \in S} \sum_{j : j \neq i} \sqrt{P_i P_j} h_i h_j K(i,j) + \sigma_n^2 \frac{1}{|S|},
\]

where \( K(i,j) \) and \( K(U,i) \) is the covariance between sensor
i and j, and the event source and sensor i, respectively.
B. Related Background in Cooperative Game Theory

The game model considered in this paper is a transferable utility game, shortly TU game, defined as follows: a TU game is a game in which the payoff of a coalition can be completely described by a single real number, characteristic function, and can be explained as the amount of payoff incurred by the cooperation of the coalition members [26]. Formally defining, characteristic function described by a single real number, is a game in which the payoff of a coalition can be completely quantified.

Definition 1: A TU game is a pair \((N, v)\) where \(N = \{1, 2, 3, ..., n\}\) is a finite set of players, i.e., coalition, and \(v(S)\) is its characteristic function \(v: 2^N \to \mathbb{R}\) that satisfies \(v(\emptyset) = 0\), and represents the total payoff that coalition \(S\) can get in the game \((N, v)\).

In cooperative game theory, core is a representative set-valued solution concept that brings more clear understanding of the implication of Shapley value. The core is regarded as a set of payoff allocations that make no player breaks away from the grand coalition, i.e., set of all players. That is, a combination of allocations is in the core if there is no subcoalition in which its members may gain a higher total outcome than the combination of allocations of the grand coalition [27].

Since the core is a set-valued solution concept, if a game has a non-empty core, the core may contain more than one solution. However, TU games have no definition of how the payoff of the grand coalition should be allocated to each player, and generally, the core is empty in many games. Therefore there is a requirement of another solution concept that makes the total payoff distributed to each player in accordance with each player’s marginal contribution to achieving the total payoff; Shapley value meets these requirements [23].

Theorem 2: For every \(i \in N\), the Shapley value \(\phi_i\) of a game \((N, v)\) is given by
\[
\phi_i (v) = \sum_{\emptyset \neq S \subseteq N \setminus \{i\}} \frac{(N - |S| - 1)!|S|!}{N!} \times \Delta_i v (S), \tag{8}
\]
where
\[
\Delta_i v (S) = v (S \cup \{i\}) - v (S). \tag{9}
\]

In Shapley value, the payoff for any player \(i\) depends on the worth of every possible coalition. It means that Shapley value premises that all permutation of the players are feasible.

III. MEASUREMENT ALLOCATION GAME

The measurement allocation problem is similar with the typical sensor selection problems in the sense that both consider the measurement of some representative sensors. However, the measurement allocation problem emphasizes on distributing measurement to the entire sensor set for balanced resource consumption even though its objective value is worse than that of the typical sensor selection problem. In addition, the uniform measurement yields the best balanced resource consumption, but it does not regard the quality of the objective value. Accordingly, by the measurement allocation, we can get in to a compromising point between the quality of the objective value and the balanced resource consumption (or network lifetime). Thus it is essential to quantify each sensor’s contribution and determine the probability of each sensor’s being measured in proportion to its contribution.

Prior to giving the game model for the measurement allocation problem, we define the measurement allocation problem as follows:

Definition 3 (Measurement allocation problem): Allocate the probability of each sensor’s being measured in proportion to each sensor’s marginal contribution to the reliable estimation of an event source in sensor field.

The estimation reliability can be expressed within the distortion error criterion in (1). Then we cast the measurement allocation problem into a cooperative game:

Definition 4 (Measurement allocation game): The measurement allocation game is then a game \((N, v)\) with the characteristic function for every coalition \(S \subseteq N\):
\[
v (S) = [D_E (S)]^{-1}. \tag{10}\]

\(v(S)\) expresses the inverse of distortion error when information given by sensors in a coalition \(S\) is correlated. Definitely, the measurement allocation game is a cooperative game with transferable utility since the payoffs of all \(S \in 2^N\) are directly expressed as \(v(S)\). While, in general cooperative games, payoff itself incurred by a cooperation is distributed to players, the payoff is the estimation reliability in the measurement allocation game. Therefore, we assume that the payoff, i.e., the estimation reliability, is converted into the measurement probability, before distributing it.

A. Shapley Value in Measurement Allocation Game

Now the Shapley value of the measurement allocation game is given by
\[
\phi_i (v) = \sum_{\emptyset \neq S \subseteq N \setminus \{i\}} \frac{(|N| - |S| - 1)!|S|!}{|N|!} \times \Delta_i v (S) \tag{11}
\]
where
\[
\Delta_i v (S) = [D_E (S \cup \{i\})]^{-1} - [D_E (\{i\})]^{-1}. \tag{12}
\]

Then the probability of each sensor’s being measured is given by
\[
\Gamma_i (v) = \frac{\phi_i (v)}{\sum_{i \in N} \phi_i (v)}. \tag{13}
\]

Due to the carrier and symmetry axioms of Shapley value [23], it gives a way of distributing a coalition payoff brought by a cooperation exactly fairly. In the measurement allocation game, it gives a way of distributing the measurement considering the correlation.

IV. RANDOMIZED METHOD

A. Algorithm

Although Shapley value has been widely studied from a theoretical point of view, the problem of its calculation was proved as a \#P-complete (Sharp-P-complete) problem [24], [25]. There have been lots of approach to compute
exact Shapley value such as multilinear extension (MLE) (also called diagonal approximation), generating functions method, decomposition method, etc. However all of them require exponential time or a large memory space. In order to overcome this intractability, a few number of approximation methods were developed, e.g., linear approximation, modified MLE, and randomized method. However, all the approximation methods except the randomized method require the exact statistic information of the characteristic function, and can be applicable to only simple cooperative games such as weighted voting game or weighted majority game [27] where a player’s marginal contribution and payoff of a coalition is a binary value. Incidentally, the MLE can be applicable to the game that has additive characteristic function. On that account, we apply the randomized method [25] in this paper. Refer [24] and references therein for more detailed explanations of Shapley value computations. Furthermore, it is known that the randomized algorithm returns good approximations to many \#P-complete problems with high probability [30].

In the randomized method, sampling and its inference are used in circumstances where it is infeasible to obtain information from every member of the original population. The randomized algorithm begin with deciding the size of permutation samples \( q_X \) for each coalition size \( X \). For this, we make a rough assumption that our characteristic function follows Gaussian normal distribution. Therefore we decide \( q_X \) with guaranteeing that the error in the estimation process is lower than \( d \) with 95% maximum allowable error as follows:

\[
q_X = \left\lfloor \frac{1.96}{d \sigma_X} \right\rfloor^2.
\]  

(14)

where \( \sigma_X \) is standard deviation estimated with small pilot samples. Then, on each coalition size \( X \), it evaluates the marginal contribution of each sensor \( i \) to the sampled coalition \( S_X \) of size \( X \); this evaluation repeats \( q_X \) times with different \( S_X \) on each repetition by

\[
\Delta_i v(S_X) = v(S_X \cup \{i\}) - v(S_X).
\]  

(15)

Conclusively, the approximate Shapley value of each sensor \( i \) is given by

\[
\hat{\varphi}_i(v) = \sum_{X=1}^{X_{\text{max}}} \left[ \frac{1}{q_X} \sum_{k=1}^{q_X} \Delta_i v(S_X^k) \right].
\]  

(16)

where \( X_{\text{max}} \) is the maximal number of sensors to be activated, and given by the sensor application.

Then the probability of each sensor’s being measured is determined by normalizing the Shapley value with its summation:

\[
\hat{\Gamma}(i) = \frac{\hat{\varphi}_i(v)}{\sum_{i \in N} \hat{\varphi}_i(v)}.
\]  

(17)

**B. Approximation Error**

Ideally, the approximate Shapley value should be evaluated by comparing it with the exact one. However this computation cannot be done due to the intractability in finding the exact value. In this paper, we exploit that the accuracy of the randomized algorithm depends on its sampling error since it depends on the measurements on samples drawn randomly. Therefore we evaluate approximate Shapley value by measuring the sampling error. The empirical standard deviation of the approximate Shapley value of sensor \( i \) for a random permutation of coalitions of size \( X \) is given by

\[
s_i(S_X, v) = \sqrt{\frac{1}{q_X} \sum_{k=1}^{q_X} [T_i - \Delta_i v(S_X^k)]^2}
\]  

(18)

with yielding an estimator of the sampling error:

\[
e_i(S_X, v) = \frac{s_i(S_X, v)}{\sqrt{q_X}}.
\]  

(19)

By the standard error propagation rules, the error in the approximate Shapley value is

\[
e(\hat{\varphi}_i(v)) = \sum_{X=1}^{X_{\text{max}}} e_i(S_X, v).
\]  

(20)

**V. Numerical Evaluation**

In this section, we use numerical results in order to evaluate our method within three performance criteria: approximation quality, impact of the coalition structure, and balancedness between low distortion error and prolonged network life time. We consider a sensor field where sensors are randomly distributed in \( 500 \times 500 \text{m}^2 \). We use the covariance model in (3) with setting \( \alpha = 0.018 \). We compute the size of permutation samples with 95% of maximum allowable error \( d \) using 100 sample pilots. Besides, we draw each sensor’s transmission power randomly in \( 100 \text{mW} \sim 2 \text{W} \). The channel attenuation is modeled as \( h_{i,j} = K_0 \cdot 10^{\beta(i,j)/10} \cdot (d_{i,j})^{-2} \) where \( K_0 = 10^4 \), \( d_{i,j} \) is the distance between \( i \) and \( j \), and \( \beta(i,j) \) is random Gaussian variables with zero mean and standard deviation equal to 6dB.
A. Approximation Quality

We illustrate the quality of the randomized method by comparing its results to the exact value and measuring the standard sampling error. For this evaluation, we set $\sigma_z^2 = \sigma_w^2 = 1.0$.

On Fig. 2, the error between the exact Shapley values and the approximate ones are compared for the sensor field with 20 sensors. It is observed that the maximum error is measured about 0.012, and in most case, measured below 0.004. In addition, as expected usually, we notice that larger allowable error yields larger sampling error.

We next evaluate the approximate Shapley value with larger set of sensors, and estimate its accuracy with the standard sampling error given by (20). The Fig. 3 plots the results and shows that the sampling error is measured as less than 0.1%.

B. Balancedness

The last set of experiments is performed in order to investigate the balancedness of each method - balancedness between average distortion error and network lifetime that is defined as the duration until all the sensor’s energy get depleted.

As done in the previous subsection, we also compare with the greedy and uniform methods: the least balanced and the most balanced. We distribute 50 sensors on the sensor field, and assume that each sensor consumes the energy equal to its transmission power on each measurement. We also assume that every sensor can be measured $150 \sim 200$ times until its energy gets depleted. We iterate the measurement process with selecting 20 sensors on each iteration according to those three selection criteria, and measure the cumulative average distortion error on each iteration\(^1\). The results are shown in Fig. 4. On each iteration, the Shapley value-based measurements select sensors according to their measurement probabilities.

It is noticed that the average distortion errors of both the greedy and Shapley value-based methods start increasing abruptly from iteration 151 and 174 respectively due to the energy depletions in the highly contributory sensors. In addition, while the lifetime of the greedy method expires at iteration 352, the lifetime of the uniform method lasts until 427. The Shapley value-based method lasts until 390. As expected, the greedy method always shows lower average distortion error.

\(^1\)On each iteration $k$, we plot the average distortion error up to iteration $k$ from iteration 0.
error than the other methods through entire iteration, and the uniform method always yields the highest. The Shapley value-based measurements yield lower average distortion error than the uniform method and longer lifetime than the greedy method, and which illustrates the balancedness of our interest.

VI. CONCLUSION

In this paper, we address the measurement allocation problem in a spatially correlated sensor field. Our main goal is to reduce the distortion error between the event source and its estimation. By the correlation nature, we model this problem into a cooperative game, and then deploy Shapley value for fair measurement allocation. The inverse of the distortion error is defined as a payoff, and the measurement probability is a reward for sensor’s contribution to reducing the distortion error. To overcome the intractability, we apply the randomized method. Since the computation of exact Shapley value is very exhaustive, we deploy the randomized method that can compute approximate Shapley value within reasonable time.

Through numerical experiments, we evaluate the randomized method by comparing the approximate Shapley value to the exact one and measuring the sampling error. Then, we evaluate our method in terms of both the network lifetime and achieved distortion error.

For the future research direction, we can utilize Shapley value in order to find each sensor’s contribution in two different measurement allocation games since both values satisfy the linearity axiom. That is, a separate game for achieving prolonged network lifetime may be considered, and then each sensor’s contribution to both metrics can be quantified using the linearity axiom. Moreover, Shapley value can be deployed to determine the quantization level of the information detected by each sensor: the higher Shapley value a sensor has, the more bits it uses for the transmission of event.

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