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Huiyuan Zhou\textsuperscript{a}, Ram M. Narayanan\textsuperscript{a}, and Ilangko Balasingham\textsuperscript{b}

\textsuperscript{a}Department of Electrical Engineering, Pennsylvania State University, University Park, PA 16802
USA

\textsuperscript{b}Department of Electronics and Telecommunications, Norwegian University of Science and Technology, Trondheim, NO-7491, Norway

ABSTRACT

This paper addresses the detection and imaging of a small tumor underneath the inner surface of the human intestine. The proposed system consists of an around-body antenna array cooperating with a capsule carrying a radio frequency (RF) transmitter located within the human body. This paper presents a modified Levenberg-Marquardt algorithm to reconstruct the dielectric profile with this new system architecture. Each antenna around the body acts both as a transmitter and a receiver for the remaining array elements. In addition, each antenna also acts as a receiver for the capsule transmitter inside the body to collect additional data which cannot be obtained from the conventional system. In this paper, the synthetic data are collected from biological objects, which are simulated for the circular phantoms using CST studio software. For the imaging part, the Levenberg-Marquardt algorithm, which is a kind of Newton inversion method, is chosen to reconstruct the dielectric profile of the objects. The imaging process involves a two-part innovation. The first part is the use of a dual mesh method which builds a dense mesh grid around in the region around the transmitter and a coarse mesh for the remaining area. The second part is the modification of the Levenberg-Marquardt method to use the additional data collected from the inside transmitter. The results show that the new system with the new imaging algorithm can obtain high resolution images even for small tumors.

Keywords: Microwave imaging, Levenberg-Marquardt algorithm, permittivity, medical imaging, image reconstruction

1. INTRODUCTION

Microwave imaging is noninvasive method for visualizing the interior of the body for clinical analysis. In comparison to conventional imaging methods, such as MRI and CT, it has great potential benefits such as low cost and low health risk due to the nonionizing nature of electromagnetic (EM) waves, which makes the early-stage tumor detection and round-the-clock monitoring possible. Both of these advantages have made microwave tomography imaging attractive for researchers in recent years [1].

Microwave imaging can be grouped into two major categories. One is the ultra-wideband (UWB) radar approach. The UWB radar method transmits a short duration pulse to identify the locations producing an increased backscatter within the imaged body [2]. The other approach is inverse scattering method. The objective here is to calculate the profile of the internal properties of a physical object from the data collected outside the object. This problem is different from direct problems. In direct problems, the electromagnetic properties and parameters of the object are known, and scattered field is the output of the system. There are several areas where inverse problems are applied, such as astronomy, remote sensing, electromagnetic scattering, computed tomography, geophysics, etc. Inverse problems are proven to be ill-posed problems in the sense of Hadamard’s characterization [3], which contains the existence, uniqueness, and stability of the solution. Solving ill-posed problems requires regularization procedures in the algorithms.

This project aims at detecting and recognizing small tumors in intestine by capsule endoscope using a microwave technique. Currently, physicians rely on the insertion of flexible tubes containing cameras to examine hard-to-reach parts
of the digestive tract. However, this technique can only receive the visual image of the tumors which located on the inner surface of intestine. Capsule endoscopes help to fill this gap with significantly less discomfort for the patient. They can also detect small tumors underneath the inner surface of intestine which cannot be detected by the visual camera. A capsule endoscope is a camera with the size and shape of a pill that is swallowed in order to visualize the gastrointestinal tract. In addition, the capsule also carries a radio frequency (RF) transmitter to communicate with around-body antennas to reconstruct the dielectric profiles and thereby recognize the tumor underneath the inner surface of intestine. This technique can accurately locate the tumors and estimate the size of the tumors to help doctors diagnose the state of the tumor and accurately locate the object during the surgery.

This paper proposes a modified Levenberg Marquardt algorithm to reconstruct dielectric profiles of objects to image and detect small tumors using this new system. This paper consists of two major parts. The first part involves generating the synthetic data by using CST microwave studio software for the two-dimensional model. The biological object is simulated by a circular phantom which is presented in [4]. In the second part, by using the data with the modified Levenberg-Marquardt algorithm, which is a kind of modified Gauss-Newton iterative method, we obtain the reconstruction of complex permittivity profile of the objects. An empirical formula based on [5] is chosen as the method for selecting the regularization parameter.

2. THEORY AND FORMULATION

2.1 Configuration

Let us consider a configuration in which an object oriented along the z axis with its cross section in the XY plane is denoted by S, which is called the image domain [3]. The object S is placed inside a homogeneous medium, which acts as a matching medium to reduce the reflections from the object. M antennas are placed around the object at a specified radius. The region where the antennas are located constitutes the measurement domain Ω. Each antenna acts as a transmitter sequentially, and the remaining M − 1 antennas act as the receivers simultaneously. Let r(x, y) be the coordinates of a spatial point in the imaging domain. Denoting the complex permittivity at point r(x, y) as ε(r), we have

\[
\varepsilon(r) = \varepsilon'(r) - j\varepsilon''(r)
\]

where \(\varepsilon'\) and \(\varepsilon''\) are the real and imaginary parts of dielectric permittivity respectively.

The dielectric profile investigated is represented by contrast function c(r), which is defined as

\[
c(r) = \begin{cases} 
\varepsilon(r) - \varepsilon_{\text{ext}} & \text{if } r \in S \\
0 & \text{if } r \notin S
\end{cases}
\]

where \(\varepsilon_{\text{ext}}\) is the exterior medium complex permittivity.

The imaging system geometry considered here is separated into two parts. The first part has the antennas arranged circularly around the object located at the center of the circular microwave scanner. When one of the antennas transmits a signal, all the other remaining antennas will act as the receivers to collect the scattered field generated by the transmitter at the same time. There are a total of L transmissions from both outside and inside the body and the number of the receivers each time is denoted by M. For the circular antenna model, the number of receivers has the relation with number of transmitters as \(L = M + 1\). Let the coordinate of the receivers be denoted by \(s_m\) where \(m = 1 \cdots M\).

Then based on the scalar wave equation and boundary conditions, it leads to the scalar electrical field integral equation (EFIE), given by
\[ e_l(r) = e_l^i(r) + \int \int_S k_0^2 c(r') e_l(r') G(r, r') dr' \]
\[ l = 1 \cdots L \]  \hspace{1cm} (3)

where \( e^i(r), e(r) \) and \( e^s(r) \) denote incident electric field, total electric field, and scattered electric field respectively. 

\( G(r, r') \) is the two dimensional free space Green’s function, given by 
\[ G(r, r') = \frac{1}{4\pi |r - r'|} \] . Then the scattered field received by the antennas is

\[ e_l^s(s_m) = \int \int_S k_0^2 c(r') e_l(r') G(s_m, r') dr' \]
\[ l = 1 \cdots L; \quad m = 1 \cdots M \]  \hspace{1cm} (4)

Equations (3) and (4) present the forward scattering problem which is used to calculate the scattered field with the information of incident field and predicted dielectric properties.

The second configuration occurs when the in-body antenna acts as transmitter and all the antennas around object act as the receivers. The scalar electrical field integral equation written as

\[ e_c(r) = e_c^i(r) + \int \int_S k_0^2 c(r') e_c(r') G(r, r') dr' \]  \hspace{1cm} (5)

\[ e_c^s(s_m) = \int \int_S k_0^2 c(r') e_c(r') G(s_m, r') dr' \]  \hspace{1cm} (6)

where the subscript \( c \) denotes the center.

### 2.2 Dual Mesh Strategy

For the special geometry of the intestine, we assume that the tumor is located near the in-body RF transmitter. The mesh grid should be denser around the RF transmitter and could be coarser in the remaining region. The first step is to localize the in-body transmitter using a lower frequency, as suggested in [6]. The second step is to discretize the image domain with the dual mesh method, which guarantees that the area centered at the location of the in-body transmitter within a fixed radius has a denser mesh grid. The remaining areas use a coarser cell size. Figure 1 shows the mesh geometry wherein the in-body transmitter is located at \((0,0)\) with a radius of 20 mm.
Fig. 1: Dual mesh diagram when the coordinate of in-body antenna is $r = (-0.02 \text{ m}, -0.02 \text{ m})$.

Based on the dual mesh, the Green’s function in EFIE and scattered field equation need to be modified. Based on [7], the discretized object EFIE is formulated as

$$
e(m) = e^i(m) - (jk^2/4) \sum_{n=1}^{N} c(n) e(n) \cdot \iint_{n \text{ cells}} H_0^{(2)}(kd)dx' dy'$$

(7)

where $d = \sqrt{(x - x')^2 + (y - y')^2}$, and (7) means that the incident field is enforced at the center of cell $m$. A simple solution is given for the integral of the zero-order Hankel function over a circular region, which is

$$
jk^2/4 \int_{0}^{2\pi} \int_{0}^{a} H_0^{(2)}(kd)d' dd' d\phi'
\left(\frac{j}{2}\right) \left[ \pi ka H_1^{(2)}(ka) - 2j \right] \quad \text{if } m = n
= \left(\frac{j \pi ka}{2}\right) J_1(ka) H_0^{(2)}(kd_{mn}) \quad \text{if } m \neq n$$

(8)
or dual mesh strategy, since the cell size has two different values, the values of $a$ are different; thus, the EFIE and (8) are modified as follows:

$$
e(m) = e^i(m) - \{(jk^2/4) \sum_{i=1}^{N_1} c(i)e(i) \cdot \iint_{N_1 \text{ cells}} H_0^{(2)}(kd) dx' dy' \
+ (jk^2/4) \sum_{j=1}^{N_2} c(j)e(j) \cdot \iint_{N_2 \text{ cells}} H_0^{(2)}(kd) dx' dy'\}$$

(9)

and

$$
(jk^2/4) \int_0^{2\pi} \int_0^\alpha H_0^{(2)}(kd) d\theta' d\phi'

= \begin{cases}
    \left(\frac{j}{2}\right) [\pi ka_1 H_1^{(2)}(ka_1) - 2j] & \text{if } m = n, \ n \in S^1 \\
    \left(\frac{j}{2}\right) [\pi ka_2 H_1^{(2)}(ka_2) - 2j] & \text{if } m = n, \ n \in S^2
\end{cases}

= \begin{cases}
    \left(\frac{j nk a_1}{2}\right) J_1(ka_1)H_0^{(2)}(kd_{mn}) & \text{if } m \neq n, \ n \in S^1 \\
    \left(\frac{j nk a_2}{2}\right) J_1(ka_2)H_0^{(2)}(kd_{mn}) & \text{if } m \neq n, \ n \in S^2
\end{cases}

(10)

where $N_1, N_2$ are the total number of small cell size and large cell size, and $N_1 + N_2 = N$. In addition, $S^1, S^2$ are the set of small cells and large cells respectively, and $S^1 \cup S^2 = S$.

2.3 Forward Problem

The forward problem is calculated during each iteration. Then based on [8], equations (9) and (10) can be expressed in discrete form as:

$$[I - G^T C] e_i = e_i^f$$

(11)

$$e_i^s = G^T E_i c$$

(12)

$G^T$ is $N \times N$ matrix of the integrated Green’s function, $G^s$ is a $M \times N$ matrix of the integrated Green’s function and $N$ is the total number of the cells. Based on [7], the integration of the Green’s function is performed over a disk instead of a square, which is determined by the square size of the cell divided by $\sqrt{3}$.

2.4 Inverse Problem

The inverse problem is mainly solved using the Levenberg-Marquardt algorithm with the help of forward problem. First, we denote the forward problem which is expressed by equation (11) and (12) as

$$O(c) = e^s$$

(13)
It is obvious that function $O$ is a complex nonlinear vector function. Equation (14) is solved in the least squared sense which is expressed as

$$Q(c) = \|O(c) - e_{\text{meas}}^s\|^2$$ (14)

For the conventional system, the scattered field data collected by the outside antenna is a $L \times M$ matrix. For the circular antenna array, $L = M + 1$, so the data matrix has dimension as $(M + 1) \times M$. For the new system, the in-body transmitter supplies additional scatter data, which is a $(M + 1)$ vector. As the result, the cost function is defined as

$$Q(c) = \|[O(c), \rho O_c(c)] - [e_{\text{meas}}^s, \rho e_{\text{meas},c}^s]\|^2$$ (15)

where $O_c(c)$ is the calculated scattered field from forward problem, and $e_{\text{meas},c}^s$ represents the measured scattered data from $(M + 1)$ antennas outside the object. Parameter $\rho$ provides the weight of the additional data. Since the in-body transmitter is closer to the small (tumor) object compared to the outside antennas, the weight factor $\rho \geq 1$.

The inverse calculation obtains the minimum value of function $Q(c)$. Based on [8], the Levenberg-Marquardt algorithm can be written as

$$[D^*D + \alpha I_N] \Delta c = D^* \Delta e^s$$ (16)

In addition, $\alpha$ is chosen using

$$\alpha = \beta \frac{\text{Trace}[D^*D]}{N} \frac{\|O(c) - e^s\|^2}{\|e^s\|^2}$$ (17)

The initial value of $\beta$ in equation (17) is chosen empirically and $\alpha$ is updated at every iteration based on the normalized field error $\|O(c) - e^s\|^2/\|e^s\|^2$ by applying the policy below:

$$\beta_+ = \begin{cases} \beta_c & \text{if } \text{err } e^s_c - \text{err } e^s < -0.1 \text{err } e^s_c \\ \frac{1}{2} \beta_c & \text{if } -0.1 \text{err } e^s_c \leq \text{err } e^s_c - \text{err } e^s \leq 0.1 \text{err } e^s_c \\ 2\beta_c & \text{if } \text{err } e^s_c - \text{err } e^s > 0.1 \text{err } e^s_c \end{cases}$$ (18)

### 3. SIMULATION RESULTS

This section includes simulation setups both in CST and MATLAB, and the simulation results. First, we introduce the synthetic scatter field data collected from CST software. The second part employs MATLAB to implement the modified Levenberg-Marquardt algorithm to obtain the permittivity profile of the object.
3.1 Simulation Setup

In this simulation, the biological object is modeled by circular phantom with real tissue dielectric properties. The circular phantom is a circular cylinder placed in the XY plane with the electrical properties constant along the z-axis. The radius of the phantom is 13 cm with two smaller circular cylinders placed inside to make it heterogeneous, as shown in Fig 2. The radii of the three inside cylinders are 1 cm and 4 cm (both shown in red-brown color) and 3 cm (shown in dark green color). The electrical properties of the 1-cm and 4-cm radius cylinders are \( \varepsilon_r = 66 \) and \( \sigma = 1.8 \text{ S/m} \), which represents the dielectric constant of intestine. For the cylinder of 3-cm radius, the properties are \( \varepsilon_r = 47 \) and \( \sigma = 0.1 \text{ S/m} \). The main region (shown in light green color) is assumed to have \( \varepsilon_r = 57 \) and \( \sigma = 0.8 \text{ S/m} \).

In the simulation, the transmitted signal is a single frequency signal every time at two frequencies of 403.6 MHz and 799.6 MHz, which is generated by a current source oriented along the z-axis with current amplitude \( I_c = 1 \text{ A} \) [4]. Using the data from the 403.6-MHz signal, we localize the position of the inside transmitter. Then, the new system combines the outside antenna and in-body transmitter operating at a higher frequency of 799.6 MHz for the second step operation. Based on [6], the incident field in each cell is represented by,

\[
e_i^t(r) = -\frac{(\omega \mu_0 I_c)}{4} H^{(2)}_0(k_{ext}|r - r_t|)
\]

where \( r \) is the coordinate of the cells, \( r_t \) is the coordinate of the transmitter, and \( k_{ext} \) is the propagation constant in the exterior medium.

All of the antennas are arranged around the circular model at a spacing of 5°; hence the total number of the outside antennas is 72, which means the number of transmitter in the entire procedure of the simulation is \( L = 72 \) and the total number of the receivers every time is \( M = 71 \). Then, the in-body transmitter sends the RF signal and all the outside antennas act as receiver. In this case, the number of receivers is 72.

During the iteration each time, we specify upper and lower boundaries for the permittivity constant and the conductivity, which are \( \varepsilon_{r\text{ max}} = 66 \), \( \varepsilon_{r\text{ min}} = 47 \) and \( \sigma_{\text{ max}} = 1.8 \text{ S/m} \), \( \sigma_{\text{ min}} = 0.1 \text{ S/m} \). This setup provides the a priori information which will lead the simulation to converge faster and remain more stable.

For the imaging part, the circular phantom is discretized with a dual mesh method with \( N \) cells. The cell size for the 403.6-MHz signal is set up around \( \lambda_{\text{ext}}/8 \), and the cell size of denser mesh grid around the in-body transmitter is about \( \lambda_{\text{ext}}/16 \) when the signal frequency is 799.6 MHz.
The region for denser mesh is chosen as a circular area with its center at the position of in-body transmitter whose radius is fixed as 2 cm. The initial values of the relative permittivity and conductivity of all cells are set the same as the exterior medium, which is \( \varepsilon_r = 5.7 \) and \( \sigma = 0.8 \) S/m. The chosen cell size is based on the trade-off between the computation time, convergence time, and resolution of the image.

### 3.2 Simulation Result and Analysis

With the modified Levenberg-Marquardt algorithm, the reconstructed complex permittivity profiles are shown in Fig 3.

![Relative Permittivity and Conductivity](image)

Fig. 3: Relative permittivity and conductivity reconstruction result of the modified algorithm.

As the coordinate of in-body antenna is estimated at \( r = (0, 0) \), the cells around the origin are \( 0.33 \times 0.33 \text{ cm}^2 \), and the cell size for other cells is \( 0.67 \times 0.67 \text{ cm}^2 \). From the reconstruction result of relative permittivity, it is possible to detect the position of the 0.5-cm radius cylinder inside the object. At the same time, the result of conductivity profile provides a better image to recognize the shape of the two bigger cylinders. In MATLAB simulation, the iteration process of the inverse problem stops when the normalized field error is smaller than 0.1. Fig 4 shows the plot of the normalized field error as a function of the number of iterations. The inverse process stops at the 17th iteration when the normalized error approach 0.08563, which is smaller than 0.1 and triggers the algorithm to stop.

![Normalized Field Error](image)

Fig. 4: Normalized field error as a function of number of iterations for the modified Levenberg-Marquardt algorithm.
In Fig 5, the results of relative permittivity of the object at different iterations are shown. It shows the result at the 1st, 5th, 10th, 15th iterations, and the final result at the 17th iteration. As shown in Fig. 5, the reconstruction result becomes better when the normalized error approaches 0.1. The dielectric profile starts from the same value as the exterior material. In the result, the major part of the object is in yellow color as the value is around 57 for the relative permittivity and 0.8 S/m for the conductivity. As the normalized error reduces, the dielectric profile of the object approaches the final reconstruction results.

Fig. 5: Relative permittivity and conductivity of the circular phantom at different iterations. (a) relative permittivity at 1st iteration; (b) conductivity at 1st iteration; (c) relative permittivity at 5th iteration; (d) conductivity at 5th iteration; (e) relative permittivity at 10th iteration; (f) conductivity at 10th iteration; (g) relative permittivity at 15th iteration; (h) conductivity at 15th iteration; (i) relative permittivity at final (17th) iteration; (j) conductivity at 17th iteration.
Fig. 5 (continued): Relative permittivity and conductivity of the circular phantom at different iterations. (a) relative permittivity at 1st iteration; (b) conductivity at 1st iteration; (c) relative permittivity at 5th iteration; (d) conductivity at 5th iteration; (e) relative permittivity at 10th iteration; (f) conductivity at 10th iteration; (g) relative permittivity at 15th iteration; (h) conductivity at 15th iteration; (i) relative permittivity at final (17th) iteration; (j) conductivity at 17th iteration.

Fig. 6: Comparison of different reconstruction approaches. (a) relative permittivity and conductivity result at 403.6-MHz frequency without in-body transmitter; (b) relative permittivity and conductivity result at 799.6-MHz frequency without in-body transmitter; (c) relative permittivity and conductivity result with in-body transmitter and modified dual mesh algorithm.
Fig 6 (a) presents the result of relative permittivity and conductivity using the 403.6-MHz signal without the in-body transmitter. The cell size of the image domain is $0.67 \times 0.67 \text{ cm}^2$. Based on result, it is possible to estimate the position of the small cylinders in the object, but impossible to tell the exactly shape of the cylinders inside the object. Fig 6 (b) shows the results of relative permittivity and conductivity using the 799.6-MHz signal without the in-body transmitter. The cell size is still setup as $0.67 \times 0.67 \text{ cm}^2$. The result is not much better than the one obtained from the 403.6-MHz signal. Since the higher frequency allows the algorithm to discretize the object into smaller cell sizes, the cell size of the area around in-body transmitter for the dual mesh case is chosen as $0.33 \times 0.33 \text{ cm}^2$. Using the in-body transmitter and the modified algorithm, Fig 6 (c) shows that it is possible to fully recover the shape of the small cylinders inside the object and also detect the small 1-cm diameter cylindrical object near the in-body transmitter. The modified algorithm combines with the dual mesh method as introduced in Section II with a coarse mesh of size $0.67 \times 0.67 \text{ cm}^2$ and a denser mesh of size $0.33 \times 0.33 \text{ cm}^2$. The dual mesh method reduces the computation time, and at the same time achieves higher resolution around the in-body transmitter. Fig 7 shows the comparison of convergence rate for the three methods.

![Fig. 7: Comparison of normalized field error for the three methods.](image)

Based on Fig 7, the iteration number to achieve a normalized error of 0.1 for the 403.6-MHz and the 799.6-MHz signals are 13 and 20, respectively. The number for the new algorithm is 17. Basically, all of the three methods have similar convergence speed. The first case with the 403.6-MHz signal has the fastest convergence speed, because of the total number of the cells is small. On other hand, the ratio between cell size and wavelength is suitable for calculation. With higher frequency, the second method converges slower. The convergence rate of the modified algorithm is slightly lower comparing to the 403.6-MHz signal but faster than the 799.6-MHz signal without in-body RF transmitter.

4. CONCLUSION

A modified Levenberg-Marquardt algorithm is applied to the new system which incorporates an in-body transmitter to obtain a better image reconstruction for small size objects, such as small tumors. The system combines the conventional antenna array around the object with an RF transmitter inside the body. The modified algorithm uses a two-step method to form the image. The first step uses the antenna around the object operating at a lower frequency to locate the in-body RF transmitter. Next, the second step operates at a higher frequency combined with the dual mesh method to obtain higher resolution images to detect small objects in the area around the inside transmitter. The synthetic data are collected under transverse magnetic (TM) illumination from the circular phantom. The results show that using the modified Levenberg-Marquardt method, it is possible to detect and image the small object near the transmitter, which is difficult to obtain from the conventional system. In addition, the convergence rate of the modified method is faster and steady compared to the
conventional algorithm with higher resolution within the area of interest. However, the algorithm used here is based on the first order Born approximation which is suitable for a weakly scattering object. In future work, the development of algorithms for strongly scattering objects will be addressed.

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